

Three Generations of SUSY Standard Model of Nambu-Goto String

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A four dimensional Superstring have been constructed starting from a twenty six dimensional bosonic string. Fermions are introduced by noting the Mandelstam's proof of equivalence of two fermions to one boson in 1+1 dimensions. The action of the superstring is invariant under $SO(6) \otimes SO(5)$ in the world sheet. It has four bosonic coordinates and forty four Majorana fermions of $SO(3, 1)$. Here the superstring action is shown to be invariant under conformal and superconformal transformations with the usual conformal and superconformal ghosts of the 10-d superstrings. The massless spectrum obtained by quantising the action, contain vector mesons which are generators of the $SO(6) \otimes SO(5)$. Using Wilson loops, this product group is proven to descend to $Z_3 \otimes SU(3) \otimes SU(2) \otimes U(1)$ without breaking supersymmetry. Thus there are just three generations of quarks and leptons.

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1. INTRODUCTION

Recently, we have shown that a novel four dimensional Superstring can be obtained from the basic Nambu-Goto bosonic string. In this paper, we report the construction of the same $N = 1$ four dimensional superstring in which the spectrum of excitations includes three generations of the Standard Model. The theory is anomaly free, modular invariant, free of ghosts and is unitary. Some salient features of the model are outlined below.

In the earlier work on this topic as contained in references [1] and [2], the importance of superconformal invariance for the superstring had not been stressed for arbitrary genus $g > 0$. We retrieve the superconformal ghosts and show how they cancel the anomaly. The normal quantum ordering constant remain the same $a = -1$, even though there are two equivalent ways of writing down the total super Virasoro energy momentum generator. This is given in Section-4. There have been many attempts to study four dimensional strings [3], specially from the later half of the eighties. Antoniadis et al [4, 5, 6] have used eighteen fermions with four bosons in trilinear coupling and Chang and Kumar with Thirring interaction. The situation and methodology, as prevailed in 1988/1989, has been given in reference [7] with many specific examples of construction.

Not long ago, Casher, Englert, Nicolai and Taormina [8] showed that consistent superstring can be solutions of $D = 26$ bosonic strings and the latter appears to be the fundamental string theory. This has been persued further in [8] and [9, 10, 11, 12]. The latest approaches have been to derive the conventional ten dimensional superstring from the 26-d bosonic string and then compactify to reach out to phenomenology. There are several $U(1)$ s. The present formulation of the theory is quite different in calculational details and results, from those given in earlier references importantly from [4] to [8]. For completeness, we review some major steps given in our earlier references.

The model, which we have proposed, begins from the Nambu-Goto [13, 14] bosonic string theory in the world-sheet (σ, τ) in 26 dimensions. The reason for the dimensionality is easy to see. The string action in twenty six dimension is

$$S_B = -\frac{1}{2\pi} \int d^2\sigma (\partial_\alpha X^\mu \partial^\alpha X_\mu), \quad \mu = 0, 1, 2, \dots, 25, \quad (1.1)$$

where $\partial_\alpha = (\partial_\sigma, \partial_\tau)$. The central charge for bosons is found by using the general expression for the two energy momentum tensors at two world sheet points z and ω

$$2 \langle T(z)T(\omega) \rangle = \frac{C}{(z - \omega)^4} + \dots \quad (1.2)$$

The coefficient of the most divergent term as C in equation (1.2) is the central charge. Methods and principles of calculation of the central charge and those for a variety of strings has been given in reference [7]. For free bosons, the central charge is

$$C_B = \delta_\mu^\mu. \quad (1.3)$$

The action S_B in equation (1.1) has a central charge $\delta_\mu^\mu=26$ and hence it is not anomaly free. One adds to equation (1.1) the action of conformal ghosts (c^+, b_{++}) and the generator with quanta (c, b) ,

$$S_{FP} = \frac{1}{\pi} \int (c^+ \partial_- b_{++} + c^- \partial_+ b_{--}) d^2\sigma, \quad L_m^{FP} = \sum_n (m-n) b_{m+n} c_{-n}. \quad (1.4)$$

This action has a central charge -26 , independent of the dimensionality of the string. To have an anomaly free string theory, the central charge of the conformal ghosts should be able to cancel and this happens when the $C = \delta_\mu^\mu = D = 26$. So the string is physical only in $D = 26$ dimensions with the total central charge zero. Using Mandelstam's [15] proof of equivalence between one boson to two fermionic modes in the infinite volume limit in 1+1 dimensional field theory, one can rewrite the action as the sum of the four bosonic coordinates X^μ of $SO(3,1)$ and forty four fermions having internal symmetry $SO(44)$, thereby loosing Lorentz invariance. If this is anomaly free, this is also true in finite intervals or a circle. Noting that the Majorana fermions can be in bosonic representation of the Lorentz group $SO(3,1)$, the forty four fermions are grouped into eleven Lorentz vectors of $SO(3,1)$ which look as a commuting internal symmetry group when viewed from the other internal quantum number space. The action is now

$$S_{FB} = -\frac{1}{2\pi} \int d^2\sigma \left[\partial^\alpha X^\mu(\sigma, \tau) \partial_\alpha X_\mu(\sigma, \tau) - i \sum_{j=1}^{11} \bar{\psi}^{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} \right], \quad (1.5)$$

and is anomaly free with S_{FP} of equation (1.4). The upper indices j, k refer to a row and that of lower to a column, and

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (1.6)$$

and

$$\rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}. \quad (1.7)$$

Dropping indices

$$\bar{\psi} = \psi^\dagger \rho^0. \quad (1.8)$$

Here ρ^α 's are imaginary, so the Dirac operators $\rho^\alpha \partial_\alpha$ are real. In this representation of Dirac algebra, the components of the world sheet spinor $\psi^{\mu,j}$ are real and they are Majorana spinors.

One has introduced an anticommuting field $\psi^{\mu,j}$ that transforms as vectors - a bosonic representation of $SO(3,1)$. ψ_A^μ maps bosons to bosons and fermions to fermions in the space-time sense. There is no clash with spin statistics theorem. Action (1.5) is a two dimensional field theory, not a field theory in space time. ψ_A^μ transforms as a spinor under the transformation of the two dimensional world sheet. The Lorentz group $SO(3,1)$ is merely an internal symmetry group as viewed in the world sheet as stated earlier. This is discussed in reference [16].

The central charge of the free fermions in action (1.5) as deduced by calculation from equation (1.2) is

$$C_F = \frac{1}{2} \delta_\mu^\mu \delta_j^j, \quad (1.9)$$

so that the total central charge of equation (1.5) is

$$C_{SB} = \delta_\mu^\mu + \delta_\mu^\mu \delta_j^j = 4 + \left(\frac{1}{2} \times 4 \times 11\right) = 26 \quad (1.10)$$

It appears that the conformal ghosts will cancel this. As a check of the equation (1.10), the central charge of a ten dimensional superstring is $10 + (\frac{1}{2} \times 10 \times 1) = 15$. This is correct.

2. SUPERSYMMETRY

The action (1.5), however, is not supersymmetric. The eleven $\psi_A^{\mu,j}$ have to be further divided into two species; $\psi^{\mu,j}$, $j = 1, 2, \dots, 6$ and $\phi^{\mu,k}$, $k = 7, 8, \dots, 11$. For the group of six, the positive and negative parts of $\psi^{\mu,j} =$

$\psi^{(+)\mu,j} + \psi^{(-)\mu,j}$, whereas for the group of five, allowed the freedom of phase of creation operators for Majorana fermions in $\phi^{\mu,k} = \phi^{(+)\mu,k} - \phi^{(-)\mu,k}$. The action is now

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[\partial_\alpha X^\mu \partial^\alpha X_\mu - i \bar{\psi}^{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} + i \bar{\phi}^{\mu,k} \rho^\alpha \partial_\alpha \phi_{\mu,k} \right]. \quad (2.1)$$

Besides $SO(3,1)$, the action (2.1) is invariant under $SO(6) \otimes SO(5)$. It is also invariant under the supersymmetric transformation

$$\delta X^\mu = \bar{\epsilon} (e^j \psi_j^\mu - e^k \phi_k^\mu), \quad (2.2)$$

$$\delta \psi^{\mu,j} = -i e^j \rho^\alpha \partial_\alpha X^\mu \epsilon, \quad (2.3)$$

$$\text{and} \quad \delta \phi^{\mu,k} = i e^k \rho^\alpha \partial_\alpha X^\mu \epsilon. \quad (2.4)$$

Here ϵ is a constant anticommuting spinor. e^j and e^k are eleven numbers of a row with $e^j e_j = 6$ and $e^k e_k = 5$. They look like $e^1 = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ and $e^7 = (0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0)$. Such arrays have been introduced in reference [7] to obtain different group structure. Ours is a specific case with phenomenological applications.

There is wide mismatch between the fermionic and bosonic modes in the action (2.1). To investigate this disturbing feature, we find that the commutators of two successive supersymmetric transformations lead to a translation with the coefficients $a^\alpha = 2 i \bar{\epsilon}^1 \rho^\alpha \epsilon_2$ provided the internal symmetry indices in equation (2.1) satisfy

$$\psi_j^\mu = e_j \Psi^\mu, \quad \text{and} \quad \phi_k^\mu = e_k \Psi^\mu. \quad (2.5)$$

These are the two key equations. These equations state that, of the eleven sites (j, k) , the super fermionic partner Ψ^μ is found in one site only. On j^{th} or k^{th} site, it emits or absorbs quanta as found by quantising $\psi^{\mu,j}$ or $\phi^{\mu,k}$ respectively prescribed by the action (2.1). The alternative auxiliary fields are not needed. The Ψ^μ is given by the linear sum,

$$\Psi^\mu = e^j \psi_j^\mu - e^k \phi_k^\mu. \quad (2.6)$$

It is easy to verify that

$$\delta X^\mu = \bar{\epsilon} \Psi^\mu, \quad \delta \Psi^\mu = -i \epsilon \rho^\alpha \partial_\alpha X^\mu \quad (2.7)$$

and

$$[\delta_1, \delta_2] X^\mu = a^\alpha \partial_\alpha X^\mu, \quad [\delta_1, \delta_2] \Psi^\mu = a^\alpha \partial_\alpha \Psi^\mu. \quad (2.8)$$

We immediately obtain a Nambu-Goto superstring in four dimensions from the action (2.1) using equation (2.5). The action is

$$S = -\frac{1}{2\pi} \int d^2\sigma \left(\partial_\alpha X^\mu \partial^\alpha X_\mu - i \Psi^\mu \rho^\alpha \partial_\alpha \Psi_\mu \right) \quad (2.9)$$

Thus the action (2.1) is truly supersymmetric and is also the equivalent action of a superstring in four dimension. Quantising this action in well written down procedure, the particle spectrum is very rich unlike quantising (2.9). We also need the internal symmetry $SO(6) \otimes SO(5)$ of the action (2.1).

3. SUPERCONFORMAL INVARIANCE

In the formulation of superstring theory in the above section, a principal role has been played by the proof that the commutator of two supersymmetric transformations gives a world sheet translation. Therefore, it is necessary to have an exact framework in which the super Virasoro conditions can emerge as gauge conditions. For this, the action given in (2.1) should incorporate the superconformal invariance of a full superstring theory.

From equations (2.7) and (2.8), it follows that the superpartner of X^μ is Ψ^μ . Introducing another supersymmetric pair, the Zweibein $e^\alpha(\sigma, \tau)$ and the gravitons $\chi_\alpha = \nabla_\alpha \epsilon$, the local 2-d supersymmetric action, first written down by Brink, Di Vecchia, Howe, Deser and Zumino [17, 18] is

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu - i \bar{\Psi}^\mu \rho^\alpha \partial_\alpha \bar{\Psi}_\mu + 2 \bar{\chi}_\alpha \rho^\beta \rho^\alpha \Psi^\mu \partial_\beta \chi_\mu + \frac{1}{2} \bar{\Psi}^\mu \Psi_\mu \bar{\chi}_\beta \rho^\beta \rho^\alpha \chi_\alpha \right]. \quad (3.1)$$

A detailed derivation is given in reference [16]. The Einstein-Hilbert action $\int e R \, d^2\sigma$ can be added, but this does not change the classical analysis. The action is invariant under local supersymmetric transformations

$$\delta X^\mu = \bar{\epsilon} \Psi^\mu, \quad \text{and} \quad \delta \Psi^\mu = -i \rho^\alpha \epsilon (\partial_\alpha X^\mu - \bar{\Psi}^\mu \chi_\alpha), \quad (3.2)$$

and

$$\delta e_\alpha^a = -2i \bar{\epsilon} \rho^a \chi_\alpha, \quad \delta \chi_\alpha = \nabla_\alpha \epsilon. \quad (3.3)$$

There are two other important transformations,

(a) the Weyl transformations

$$\delta X^\mu = 0, \quad \delta \Psi^\mu = -\frac{1}{2} \Lambda \Psi^\mu, \quad (3.4)$$

$$\delta e_\alpha^a = \Delta e_\alpha^a, \quad \text{and} \quad \delta \chi_\alpha = \frac{1}{2} \Lambda \chi_\alpha, \quad (3.5)$$

and

(b) the local fermionic symmetry, with η , an arbitrary Majorana spinor

$$\delta \chi_\alpha = i \rho_\alpha \eta, \quad (3.6)$$

and

$$\delta e_\alpha^a = \delta \psi^\mu = \delta X^\mu = 0. \quad (3.7)$$

The invariance (b) requires the identity $\rho^\alpha \rho_\beta \rho_\alpha = 0$, which is true in two dimensions. All these transformation properties imply that the action(3.1) is superconformal invariant. Varying the field and Zweibein, the Noether current J^α and the energy momentum tensor $T_{\alpha\beta}$ vanishes,

$$J_\alpha = \frac{\pi}{2e} \frac{\delta S}{\delta \chi^\alpha} = \rho^\beta \rho_\alpha \bar{\Psi}^\mu \partial_\beta X_\mu = 0, \quad (3.8)$$

and

$$T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{i}{2} \bar{\Psi}^\mu \rho_{(\alpha} \partial_{\beta)} \Psi_\mu = 0. \quad (3.9)$$

These are the super Virasoro constrain equations as derived from the algebra.

In a light cone basis, the vanishing of the lightcone components are obtained from variation of the action(3.1) i.e. equations (3.8) and (3.9)

$$J_\pm = \partial_\pm X_\mu \Psi_\pm^\mu = 0, \quad (3.10)$$

and

$$T_{\pm\pm} = \partial_\pm X^\mu \partial_\pm X_\mu + \frac{i}{2} \psi_\pm^{\mu j} \partial_\pm \psi_{\pm\mu, j} - \frac{i}{2} \phi_\pm^{\mu k} \partial_\pm \phi_{\pm\mu, k}, \quad (3.11)$$

where $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$.

The action in equation (3.1) is not space-time supersymmetric. However, in the fermionic representation $SO(3,1)$, fermions are Dirac spinor with four components α . We construct Dirac spinor, like equation (2.6), as the sum of component spinors

$$\Theta_\alpha = \sum_{j=1}^6 e^j \theta_{j\alpha} - \sum_{k=7}^{11} e^k \theta_{k\alpha}. \quad (3.12)$$

With the usual Dirac matrices Γ^μ , since the identity

$$\Gamma_\mu \psi_{[1} \bar{\psi}_2 \Gamma^\mu \psi_{3]} = 0, \quad (3.13)$$

is satisfied due to the Fierz transformation in four dimension, the Green-Schwarz action [20] for $N = 1$ supersymmetry is

$$S = \frac{1}{2\pi} \int d^2\sigma \left(\sqrt{g} g^{\alpha\beta} \Pi_\alpha \Pi_\beta + 2i\epsilon^{\alpha\beta} \partial_\alpha X^\mu \bar{\Theta} \Gamma_\mu \partial_\beta \Theta \right), \quad (3.14)$$

where

$$\Pi_\alpha^\mu = \partial_\alpha X^\mu - i\bar{\Theta} \Gamma^\mu \partial_\alpha \Theta. \quad (3.15)$$

This is the $N = 1$ and $D = 4$ superstring, originating from the $D = 26$ bosonic string. It is difficult to quantise this action covariantly. It is better to use NS-R [21, 22] formulation with G.S.O projection [23]. This has been done in references [1, 2].

4. SUPERCONFORMAL GHOSTS

In a conformal ghost space equation(1.4), the path integration over metric $h_{\alpha\beta}$ could be simply replaced by one over conformal factor ϕ and reparametrisation coordinates. This is true only if the given metric can be reached by such inputs. For world sheets of genus $g > 0$, it is not possible to do so and reach all metrics from just one in this simple way. Something more must be done using superconformal transformations.

The anticommuting ghost coordinates are present in this superstring. But the bosons are less and fermions more. These are more intricate ones due to the local world sheet supersymmetry and superconformal invariance.

So we should attempt to isolate this superconformal ghost action following well laid down procedure for local world sheet supersymmetry. Actually, the gravitino χ_α has been gauged away. So this string is anomaly free, even without them. We shall put it back in the path integral and retrieve the superconformal ghost action. Using η and ϵ as in equations (3.6) and (3.7),

$$\chi_\alpha = i\rho_\alpha \eta + \nabla_\alpha \epsilon. \quad (4.1)$$

The integration variable has been changed to that of η and ϵ ,

$$\delta\chi_{3/2} = \nabla_1 \epsilon_{1/2}, \quad \delta\chi_{1/2} = \nabla_1 \epsilon_{-1/2} + \eta_{1/2}, \quad (4.2)$$

and

$$\delta\chi_{-1/2} = \nabla_{-1} \epsilon_{1/2} + \eta_{-1/2}, \quad \text{and} \quad \delta\chi_{-3/2} = \nabla_{-1} \epsilon_{-1/2}. \quad (4.3)$$

Two of the above four are important. Changing variables from $\chi_{3/2}$ to $\epsilon_{1/2}$, the Jacobian is [16]

$$J_{3/2} = \det^{-1} \nabla_1^{1/2 \rightarrow 3/2} = \int D\gamma_{1/2} D\beta_{-3/2} \exp \left(-\frac{1}{\pi} \int d^2\sigma \beta_{-3/2} \nabla_1 \gamma_{1/2} \right), \quad (4.4)$$

and from $\chi_{-3/2}$ to $\epsilon_{-1/2}$, the other Jacobian is,

$$J_{-3/2} = \int D\gamma_{-1/2} D\beta_{3/2} \exp \left(-\frac{1}{\pi} \int d^2\sigma \beta_{3/2} \nabla_{-1} \gamma_{-1/2} \right). \quad (4.5)$$

The resulting ghost action is [16]

$$S_{SC} = -\frac{1}{2\pi} \int d^2\sigma e h^{\alpha\beta} \bar{\gamma} \nabla_\alpha \beta_\beta. \quad (4.6)$$

The energy momentum tensor is

$$T_{++} = -\frac{1}{4} \gamma \partial_+ \beta - \frac{3}{4} \beta \partial_+ \gamma. \quad (4.7)$$

This constitutes the central charge 11 (or -11). After the usual commutator quantisation, the superconformal ghost generator is

$$L_m^{gh,sc} = \sum \left(\frac{1}{2} m + n \right) : \beta_{m-n} \gamma_n : . \quad (4.8)$$

Superconformal ghosts are necessary to build BRST physical states. With the appearance of these new ghosts, anomaly cancellation becomes subtler. The covariant formulation result for the energy momentum correlation for one loop can be put in the form

$$2 \langle T_+(\sigma) T_+(\sigma') \rangle = \frac{1 - 3k^2}{(\sigma - \sigma')^4}, \quad (4.9)$$

so that the central charge for the one loop particle is $C = \pm(1 - 3k^2)$, depending on its statistics. Here k can be related to the conformal dimension $J = (1 + k)/2$. For each boson $k = 0$ and $C = 1$, for conformal ghost $k = 3$ and $C = -26$, and for the superconformal ghost $k = 2$ and $C = 11$ due to statistics. So the central charge of the four bosons of the action (2.1) is 4, of the conformal ghost of equation (1.4) is -26 and of the superconformal ghost (4.6) is 11. All these add up to $4 - 26 + 11 = -11$. The 44 fermions of the action (2.1) have been left out. To cancel this anomaly, each fermionic loop is to be characterised by $k = \frac{1}{\sqrt{6}}$, so that the central charge for each one of a pair is $1/4$ and hence the total contribution of the 44 fermionic mode is equal to 11. Thus there is a total cancellation of all anomalies and, the superstring is anomaly free. Due to the gravitino consideration, the action (2.1) of this novel superstring has central charge $4 + 11 = 15$, same as that of the 10-d superstring, i.e., $10 + 5 = 15$.

Interestingly, this string theory falls into $N = 1$ supersymmetry as per the table of the complete list given by Polchinski [19], since $C^{gh} = -15$. It is also appropriate to consider the gravitinos to have been gauged away and their contribution to the loop integral can be included due to the 44 Mandelstam Majorana fermions. As they are normal fermions, $k = 0$ or $J = 1/2$, the 22 pairs contribute 22 to the central charge. Together with 4 bosons, the central charge is 26, the same as that of the Nambu-Goto string. We shall follow this alternative as they are near-identical descriptions. The sum of all super Virasoro generators without any anomaly is expressed in the following equations.

$$L_m^{sum} = L_m^{FP} + \frac{1}{\pi} \int_{-\pi}^{\pi} e^{im\sigma} T_{++} d\sigma \quad (4.10)$$

$$= L_m^{FP} + L_m^{gh,sc} + \frac{1}{\pi} \int_{-\pi}^{\pi} e^{im\sigma} (T_{++} - k \partial_+ T_+^F) d\sigma, \quad (4.11)$$

where

$$T_+^F = \frac{i}{2} \left(\psi_+^{j,\mu} \psi_{+, \mu} - \phi_+^{k,\mu} \phi_{+, \mu} \right). \quad (4.12)$$

The Majorana fermions are distinctly different with labels j or k . The coefficient k of the total derivative of the last term in equation (4.11) is $\frac{1}{\sqrt{6}}$. $J = (1 + \frac{1}{\sqrt{6}})$. The normal ordering constant a can be calculated from the values for single bosonic and fermionic degrees of freedom [16] and in either case $a = -1$.

5. VIRASORO GENERATORS AND PHYSICAL STATES

For a brief outline, let L_m , G_r and F_m be the Super Virasoro generators of energy, momenta and currents. Let α 's denote the quanta of X^μ fields, b 's and b' 's denote quanta of ψ and ϕ fields in NS formulation and d , d' in R formulation. Then,

$$\begin{aligned} L_m &= \frac{1}{\pi} \int_{-\pi}^{\pi} d\sigma e^{im\sigma} T_{++} \\ &= \frac{1}{2} \sum_{-\infty}^{\infty} : \alpha_{-n} \cdot \alpha_{m+n} : + \frac{1}{2} \sum_{r \in z + \frac{1}{2}} (r + \frac{1}{2}m) : (b_{-r} \cdot b_{m+r} - b'_{-r} \cdot b'_{m+r}) :, & NS \\ &= \frac{1}{2} \sum_{-\infty}^{\infty} : \alpha_{-n} \cdot \alpha_{m+n} : + \frac{1}{2} \sum_{n=-\infty}^{\infty} (n + \frac{1}{2}m) : (d_{-n} \cdot d_{m+n} - d'_{-n} \cdot d'_{m+n}) :, & R \end{aligned} \quad (5.1)$$

$$G_r = \frac{\sqrt{2}}{\pi} \int_{-\pi}^{\pi} d\sigma e^{ir\sigma} J_+ = \sum_{n=-\infty}^{\infty} \alpha_{-n} \cdot (e^j b_{n+r,j} - e^k b'_{n+r,k}), \quad NS \quad (5.2)$$

and

$$F_m = \sum_{-\infty}^{\infty} \alpha_{-n} \cdot (e^j d_{n+m,j} - e^k d'_{n+m,k}). \quad R \quad (5.3)$$

and satisfy the super Virasoro algebra with central charge $C = 26$ for the action of equation (2.1),

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{C}{12}(m^3 - m)\delta_{m,-n}, \quad (5.4)$$

$$[L_m, G_r] = \left(\frac{1}{2}m - r\right)G_{m+r}, \quad NS \quad (5.5)$$

$$\{G_r, G_s\} = 2L_{s+r} + \frac{C}{3}\left(r^2 - \frac{1}{4}\right)\delta_{r,-s}, \quad (5.6)$$

$$[L_m, F_n] = \left(\frac{1}{2}m - n\right)F_{m+n}, \quad R \quad (5.7)$$

$$\{F_m, F_n\} = 2L_{m+n} + \frac{C}{3}(m^2 - 1)\delta_{m,-n}, \quad m \neq 0. \quad (5.8)$$

Equations (5.6) and (5.8) can be obtained using Jacobi identity.

This is also known that the normal ordering constant of L_o is equal to one and we define the physical states, satisfying

$$(L_o - 1)|\phi\rangle = 0, \quad L_m|\phi\rangle = 0, \quad G_r|\phi\rangle = 0 \quad \text{for } r, m > 0, \quad NS \text{ Bosonic} \quad (5.9)$$

$$L_m|\psi\rangle = F_m|\psi\rangle = 0, \quad \text{for } m > 0, \quad :R \text{ Fermionic} \quad (5.10)$$

$$\text{and } (L_o - 1)|\psi\rangle_\alpha = (F_o^2 - 1)|\psi\rangle_\alpha = 0. \quad (5.11)$$

So we have,

$$(F_o + 1)|\psi_+\rangle_\alpha = 0 \quad \text{and} \quad (F_o - 1)|\psi_-\rangle_\alpha = 0. \quad :R \quad (5.12)$$

These conditions shall make the string model ghost free. It can be seen in a very simple way. Applying L_o condition, the state $\alpha_{-1}^\mu|0, k\rangle$ is massless. The L_1 constraint gives the Lorentz condition $k^\mu|0, k\rangle = 0$, implying a transverse photon and with $\alpha_{-1}^0|\phi\rangle = 0$ as Gupta-Bleuler condition. Applying L_2, L_3, \dots , constraints, one obtains $\alpha_m^0|\phi\rangle = 0$. Further, since $[\alpha_{-1}^0, G_{r+1}]|\phi\rangle = 0$, we have $b_{r,j}^0|\phi\rangle = 0$ and $b_{r,k}^0|\phi\rangle = 0$. All the time components are eliminated from Fock space.

6. BRST CHARGE, TACHYONLESSNESS AND MODULAR INVARIANCE

In order to prove the nilpotency of BSRT charge, we note that the conformal dimension of γ is '1/2' and that of β is '3/2', which can be deduced from equation (4.8). We can now proceed to write the nilpotent BRST charge. The part of the charge which comes from the usual conformal Lie algebra technique, is

$$(Q_1)^{NS,R} = \sum (L_{-m}c_m)^{NS,R} - \frac{1}{2} \sum (m-n) : c_{-m}c_{-n}b_{m+n} : - a c_0; \quad Q_1^2 = 0 \quad \text{for } a = 1. \quad (6.1)$$

Using the graded Lie algebra, we get the additional BRST charge, in a straight forward way, in NS and R,

$$Q'_{NS} = \sum G_{-r}\gamma_r - \sum \gamma_{-r}\gamma_{-s}b_{r+s}, \quad (6.2)$$

$$Q'_R = \sum F_{-m}\gamma_m - \sum \gamma_{-m}\gamma_{-n}b_{n+m}, \quad (6.3)$$

$$\text{and } Q_{BRST} = Q_1 + Q', \quad (6.4)$$

is such that $Q_{BRST}^2 = 0$ in both the NS and the R sector [1]. In proving $\{Q', Q'\} + 2\{Q_1, Q'\} = 0$, we have used the Fourier transforms, the wave equations and integration by parts such that

$$\sum_r \sum_s r^2 \gamma_r \gamma_s \delta_{r,-s} = \sum_r \sum_s \gamma_r \gamma_s \delta_{r,-s} = 0. \quad (6.5)$$

Thus the theory is unitary and ghost free. There are no harmful effective tachyons in the model, eventhough

$$\alpha' M^2 = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, \dots, NS \quad (6.6)$$

and

$$\alpha' M^2 = -1, 0, 1, 2, 3, \dots, R. \quad (6.7)$$

The G.S.O. projection eliminates the half integral values. The tachyonic self energy of bosonic sector $\langle 0|(L_o-1)^{-1}|0\rangle$ is cancelled by $-\langle 0|(F_o+1)^{-1}(F_o-1)^{-1}|0\rangle_R$, the negative sign being due to the fermionic loop. Such tadpole cancellations have been noted also by Chattaraputi et al in reference [12]. One can proceed a step further and write down the world sheet supersymmetric charge

$$Q = \frac{i}{\pi} \int_0^\pi \rho^0 \rho^{\dagger\alpha} \partial_\alpha X^\mu \Psi_\mu d\sigma, \quad (6.8)$$

and find, as it must,

$$\sum \{Q_\alpha^\dagger, Q_\alpha\} = 2H \quad \text{and} \quad \sum_\alpha |Q_\alpha|\phi_o\rangle|^2 = 2\langle\phi_o|H|\phi_o\rangle. \quad (6.9)$$

The ground state is of zero energy. There are no overall tachyons in this Superstring.

To prove the modular invariance, one has to use the G.S.O. condition. In covariant formulation, the number of degrees of freedom of fermions is the number obtained after subtraction of constraints from the total number. In our case, the total number is 44 and there are four constraints. So the physical fermionic modes are 40. The partition function Z can be found by putting the 8 such fermions (2^3 of $SO(6)$) in each of the five boxes. This is a multiplication of spin structure of eight fermions A_8 , five times,

$$A_8(\tau) = (\Theta_3(\tau)/\eta(\tau))^4 - (\Theta_2(\tau)/\eta(\tau))^4 - (\Theta_4(\tau)/\eta(\tau))^4, \quad (6.10)$$

and

$$A_8(1+\tau) = -A(\tau) = A(-\frac{1}{\tau}), \quad (6.11)$$

where the $\Theta(\tau)$'s are the Jacobi theta function and $\eta(\tau)$ is Dedekind eta function. Due to the Jacobi relation, $A_8(\tau) = 0$, the entire partition function, which is the product of all constituent partition functions [2] of the model, vanishes. Thus all the criteria prescribed in standard references are satisfied for this superstring.

7. GAUGE SYMMETRY AND THE SUSY STANDARD MODEL

The zero mass particle spectrum of the quantised action(2.1) is very large. They are scalars, vectors and tensorial in nature. In reference [24] with Maharana, we have shown that the massless excitations of the standard model can be found from this model. There are also the graviton and the gravitino [25]. For the gauge symmetry, one has to find massless vector bosons which are generators of a group. We follow the work of Li [26].

Consider $O(n)$. There are $\frac{1}{2}n(n-1)$ generators represented by

$$L_{ij} = X_i \frac{\partial}{\partial X_j} - X_j \frac{\partial}{\partial X_i}, \quad i, j = 1, \dots, n. \quad (7.1)$$

Using

$$\left[\frac{\partial}{\partial X_i}, X_j \right] = \delta_{ij},$$

the Lie algebra, which is the commutator relation among the generators is

$$[L_{ij}, L_{kl}] = \delta_{jk} L_{il} + \delta_{il} L_{jk} - \delta_{ik} L_{jl} - \delta_{jl} L_{ik}. \quad (7.2)$$

Hence one must have $\frac{1}{2}n(n-1)$ vector gauge bosons W_{ij}^μ with the transformation law

$$W_{ij}^\mu \rightarrow W_{ij}^\mu + \epsilon_{ik} W_{kj}^\mu + \epsilon_{jl} W_{li}^\mu, \quad W_{ji}^\mu = -W_{ij}^\mu, \quad (7.3)$$

where $\epsilon_{ij} = -\epsilon_{ji}$ are the infinitesimal parameters which characterise such rotation in $O(n)$. Under gauge transformation of second kind,

$$W_{ij}^\mu \rightarrow W_{ij}^\mu + \epsilon_{ik} W_{kj}^\mu + \epsilon_{jl} W_{li}^\mu + \frac{1}{g} \partial^\mu \epsilon_{ij}. \quad (7.4)$$

The Yang-Mills Lagrangian is then written as

$$L = -\frac{1}{4}|F_{ij}^{\mu\nu}|^2, \quad (7.5)$$

with

$$F_{ij}^{\mu\nu} = \partial^\mu W_{ij}^\nu - \partial^\nu W_{ij}^\mu + g \left(W_{ik}^\mu W_{kj}^\nu - W_{ik}^\nu W_{kj}^\mu \right), \quad (7.6)$$

$F_{ij}^{\mu\nu}$ has the obvious properties, namely,

$$\square F_{ij}^{\mu\nu} = 0, \quad \partial_\mu F_{ij}^{\mu\nu} = \partial_\nu F_{ij}^{\mu\nu} = 0, \quad F_{ij}^{\mu\mu} = 0. \quad (7.7)$$

There are two sets of field strength tensors which are found in the model, one for $SO(6)$ and the other for $SO(5)$. Since $\square F_{ij}^{\mu\nu} = 0$, one can take plane wave solution and write

$$F_{ij}^{\mu\nu}(x) = F_{ij}^{\mu\nu}(p) e^{ipx}, \quad (7.8)$$

so that,

$$p^2 F_{ij}^{\mu\nu}(p) = p_\mu F_{ij}^{\mu\nu}(p) = p_\nu F_{ij}^{\mu\nu}(p) = F_{ij}^{\nu\nu}(p) = 0 \quad \text{and} \quad F_{ij}^{\mu\nu}(p) = -F_{ji}^{\mu\nu}(p). \quad (7.9)$$

These are physical state conditions (5.9)-(5.11) as well.

$$L_0 F_{ij}^{\mu\nu}(p) = 0, \quad G_{\frac{1}{2}} F_{ij}^{\mu\nu}(p) = 0, \text{ and } \quad L_1 F_{ij}^{\mu\nu}(p) = 0. \quad (7.10)$$

The field strength tensor, satisfying (7.10), is found to be

$$F_{ij}^{\mu\nu}(p) = b_i^{\mu\dagger} b_j^{\nu\dagger} |0, p\rangle + \epsilon_{ij} (p^\mu \alpha_{-1}^\nu - p^\nu \alpha_{-1}^\mu) |0, p\rangle, \quad (7.11)$$

with $\epsilon_{ij} = e_i^\alpha e_j^\beta \epsilon_{\alpha\beta}$, $(i, j)=1, \dots, 6$ for $O(6)$ and $1, \dots, 5$ for $O(5)$ with b' replaced by b . For simplicity, we drop the \dagger 's. In terms of the excitation quanta of the string, the vector generators are

$$W_{ij}^\mu = \frac{1}{\sqrt{2ng}} n_\kappa \epsilon^{\kappa\mu\nu\sigma} b_{\nu,i} b_{\sigma,j} + \epsilon_{ij} \alpha_{-1}^\mu, \quad (7.12)$$

where n_κ is the time-like four vector and can be taken as $(1,0,0,0)$. One finds that

$$\begin{aligned} \partial^\mu W_{ij}^\nu - \partial^\nu W_{ij}^\mu &= p^\mu W_{ij}^\nu - p^\nu W_{ij}^\mu \\ &= \frac{1}{\sqrt{2ng}} (n_\kappa \epsilon^{\kappa\nu\lambda\sigma} p^\mu - n_\kappa \epsilon^{\kappa\mu\lambda\sigma} p^\nu) b_{\lambda,i} b_{\sigma,j} + \epsilon_{ij} (p^\mu \alpha_{-1}^\nu - p^\nu \alpha_{-1}^\mu). \end{aligned} \quad (7.13)$$

As μ must be equal to ν , if κ, λ and σ are the same, the first term vanishes and

$$\begin{aligned} g \left(W_{ik}^\mu W_{kj}^\nu - W_{ik}^\nu W_{kj}^\mu \right) &= \frac{1}{2n} \left(n_\kappa \epsilon^{\kappa\mu\lambda\sigma} b_{\lambda,i} b_{\sigma,k} n_{\kappa'} \epsilon^{\kappa'\nu\lambda'\sigma'} b_{\lambda',k} b_{\sigma,j} - \mu \leftrightarrow \nu \right) + \epsilon_{ik} \epsilon_{kj} [\alpha_{-1}^\mu, \alpha_{-1}^\nu] \\ &= b_i^\mu b_j^\nu. \end{aligned} \quad (7.14)$$

We have used

$$\{b_{\lambda'k}, b_{\sigma k}\} = \eta_{\lambda'\sigma} \delta_{kk} = n \eta_{\lambda'\sigma},$$

and the creation operators $\alpha_{-1}^\mu, \alpha_{-1}^\nu$ commute. Equation (7.11) is referred as the field strength. Since the product of pairs of b and b' commute, the gauge group of the action (2.1) is the product group $SO(6) \otimes SO(5)$. This is same as the symmetry group of the action (2.1).

To descend to the standard model group $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$, one normally introduced by Higgs which break gauge symmetry and supersymmetry. However, if one uses the method of symmetry breaking by using Wilson's loops, supersymmetry remains intact but the gauge symmetry is broken. The Wilson loop is

$$U_\gamma = P \exp \left(\oint_\gamma A_\mu dx^\mu \right). \quad (7.15)$$

P represents the ordering of each term with respect to the closed path γ . $SO(6)=SU(4)$ descends to $SU_C(3) \otimes U_{B-L}(1)$. This breaking can be accomplished by choosing one element of U_0 of $SU(4)$, such that

$$U_0^2 = 1. \quad (7.16)$$

The element generates the permutation group Z_2 . Thus

$$\frac{SO(6)}{Z_2} = SU_C(3) \otimes U_{B-L}(1), \quad (7.17)$$

without breaking supersymmetry. Similarly, $SO(5) \rightarrow SO(3) \otimes SO(2) = SU(2) \otimes U(1)$. We have,

$$\frac{SO(5)}{Z_2} = SU(2) \otimes U(1). \quad (7.18)$$

Thus

$$\frac{SO(6) \otimes SO(5)}{Z_2 \otimes Z_2} = SU_C(3) \otimes U_{B-L}(1) \otimes U_R(1) \otimes SU_L(2), \quad (7.19)$$

making an identification with the usual low energy phenomenology. But this is not the standard model. We have an additional $U(1)$. However, there is an instance in E_6 , where there is a reduction of rank by one and several $U(1)$'s. Following the same idea [16], we may take

$$U_\gamma = (\alpha_\gamma) \otimes \begin{pmatrix} \beta_\gamma & & \\ & \beta_\gamma & \\ & & \beta_\gamma^{-2} \end{pmatrix} \otimes \begin{pmatrix} \delta_\gamma & \\ & \delta_\gamma^{-1} \end{pmatrix}. \quad (7.20)$$

$\alpha_\gamma^3 = 1$ such that α_γ is the cube root of unity. This structure lowers the rank by one. We have,

$$\frac{SO(6) \otimes SO(5)}{Z_3} = SU_C(3) \otimes SU_L(2) \otimes U_Y(1), \quad (7.21)$$

and find the supersymmetric standard model.

We now elaborately discuss the Z_3 , described by [27],

$$g(\theta_1, \theta_2, \theta_3) = \left(\frac{2\pi}{3} - 2\theta_1, \frac{2\pi}{3} + \theta_2, \frac{2\pi}{3} + \theta_3 \right). \quad (7.22)$$

For the first Wilson loop, the angle integral for $\theta_1 = \frac{2\pi}{9}$ to $\frac{2\pi}{3} - \frac{4\pi}{9} = \frac{2\pi}{9}$, so that the first loop integral vanishes. $\theta_2 = 0$ to $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$ for the second loop, is described by a length parameter R , $\theta_3 = 0$ to $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$ for the remaining loop with the same R . We take the polar components of the gauge fields as non-zero constants, as given below.

$$gA_{\theta_2}^{15} = \vartheta_{15}$$

for $SO(6) = SU(4)$ for which the diagonal generator t_{15} breaks the symmetry and

$$g'A_{\theta_3}^{10} = \vartheta'_{10}$$

for $SO(5)$, the diagonal generator being t'_{10} . The generators of both $SO(6)$ and $SO(5)$ are 4×4 matrices. We can write the Z_3 group as

$$T = T_{\theta_1} T_{\theta_2} T_{\theta_3}. \quad (7.23)$$

We have $T_{\theta_1} = 1$. This leaves the unbroken symmetry $SU(3) \times SU(2)$ untouched

$$T_{\theta_2} = \exp \left(i t_{15} \int_0^{\frac{4\pi}{3}} \vartheta_{15} R d\theta_2 \right). \quad (7.24)$$

$T_{\theta_2} \neq 1$ breaks the $SU(4)$ symmetry. Again,

$$T_{\theta_3} = \exp \left(i t'_{10} \int_0^{\frac{4\pi}{3}} \vartheta'_{10} R d\theta_3 \right). \quad (7.25)$$

$T_{\theta_3} \neq 1$ breaks the $SO(5)$ symmetry. But the remaining product of Z_3 is

$$T_{\theta_2} T_{\theta_3} = \exp \left(i \int_0^{\frac{4\pi}{3}} (\vartheta'_{10} t'_{10} + \vartheta_{15} t_{15}) R d\theta \right). \quad (7.26)$$

ϑ_{15} and ϑ'_{10} are arbitrary constants. We can choose them in such a way that

$$t_{15} \vartheta_{15} + t'_{10} \vartheta'_{10} = 0, \frac{3}{2R}, \dots$$

The term in the exponential is zero or multiples of $2\pi i$. Thus $T = U(1)$ and equation (7.18) is obtained, reducing the rank by one.

8. REPLICATION OF FAMILIES AND CONCLUDING REMARKS

The residual supersymmetry at electroweak scale is $Z_3 \times SU_C(3) \times SU_L(2) \times U_Y(1)$ and will be denoted by GZ. To find the number of generations of the theory n_G , we have to calculate Euler number χ , since

$$n_G = \frac{1}{2} \chi(GZ). \quad (8.1)$$

The following formulas are relevant. The space with known Euler number is

$$CP_N = \frac{SU(N+1)}{U(N)} \quad (8.2)$$

with

$$\chi(CP_N) = (N+1). \quad (8.3)$$

Using the the above equations (8.2) and (8.3), we find $\chi(GZ) = 6$ and $n_G=3$. These group structure gives the number of generations to be 3. This number is topological because of Dirac index theorem. The Dirac equation, in general, can have zero modes,

$$\gamma \cdot p \psi = \not{p} \psi = \not{D} \psi = 0. \quad (8.4)$$

The index of this operator is equal to the difference between the positive, n_+ and negative, n_- chiralities of the zero modes. The exact theorem, on its relation with n_G , is

$$n_G = \frac{1}{2} \chi = \text{index}(\not{D}) = n_+ - n_-, \quad (8.5)$$

of the zero modes. It is necessary to find the massless four dimensional Dirac spinors. The creation operators of the Ramond sector is

$$D^{\mu\dagger} = e^j d_{-1,j} - e^k d_{-1,k}, \quad (8.6)$$

is such that a zeromass spinorial state is [25]

$$\phi_{o+} = D_{-1}^{\mu} |0\rangle > u_{\mu} = D^{\mu\dagger} |0\rangle > u_{\mu}, \quad (8.7)$$

u_{μ} is a spinor four vector. Similarly, there is another state

$$\phi_{o-} = \alpha_{-1}^{\mu} |0\rangle > v_{\mu} = \alpha^{\mu\dagger} |0\rangle > v_{\mu}, \quad (8.8)$$

for the coordinate excitation of the Ramond sector, v_{μ} is also another four vector spinor. Both (u_{μ}, v_{μ}) are in four dimensions. They can be distinguished by (2.4)

$$\gamma_5 u_{\mu} = u_{\mu}, \quad (8.9)$$

$$\text{and} \quad \gamma_5 v_{\mu} = -v_{\mu}. \quad (8.10)$$

Together, they can also be considered as a four component spin vector ψ_μ . Since F_o is essentially the Dirac gamma matrix (γ), the condition $F_o\psi_\mu$ gives the zeromass spinor equation

$$\gamma \cdot p \psi_\mu = 0. \quad (8.11)$$

The states $\phi_{0,\pm}$ contains not only a spin- $\frac{1}{2}$ but also a spin- $\frac{3}{2}$ state. But one can covariantly separate out the Dirac spin- $\frac{1}{2}$ equation as given in [23]. If ψ_{Dirac} is any Dirac spinor in 4-dimensions satisfying $\not{p}\psi=0$, then the spin- $\frac{1}{2}$ component is

$$\psi_\mu^{\frac{1}{2}} = \frac{1}{2} [\gamma_\mu - p_\mu(\gamma \cdot \bar{p})] \psi_{Dirac}, \quad (8.12)$$

where the momentum \bar{p}_μ is conjugate to p_μ with $p^2 = \bar{p}^2 = 0$ and $p \cdot \bar{p} = 1$.

The zero mass modes of Dirac objects in standard model are grouped into three families(generations). They are

$$\begin{pmatrix} u \\ d \\ \nu_e \\ e \end{pmatrix}, \begin{pmatrix} c \\ s \\ \nu_\mu \\ \mu \end{pmatrix}, \quad and \quad \begin{pmatrix} t \\ b \\ \nu_\tau \\ \tau \end{pmatrix} \quad (8.13)$$

The left handed ones are doublets and right handed ones are singlets. The quarks are colored. There are several fermionic zero modes, 24 are with +ve helicity and 21 with negative helicity. Thus $n_+ - n_- = 3$ as per topological findings. So there have to be only three fermions(neutrinos with unpaired helicity) in the standard model.

Thus, we have made the successful attempt in constructing an anomaly free $N = 1, D = 4$ superstring from bosonic string with the gauge symmetry $SO(6) \otimes SO(5)$ which, with the help of Wilson loops, descend to the SUSY standard model. There are just three generations. This is a very important result which can help not only in phenomenology but also in finding the correct dynamics of interacting string following up the bosonic equivalence.

It has been summarised by Mohapatra [28] that the uniqueness of the three generations model constructed by Tian and Yau, is the triumph of Calabi-Yau compactification procedure and all such models are equivalent. We propose to study if the present model also falls in the same category.

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